Notes
Chapter – 13
Direct and Inverse Proportions

- **Variations:** If the values of two quantities depend on each other in such a way that a change in one causes corresponding change in the other, then the two quantities are said to be in variation.
- **Direct Variation or Direct Proportion:**
  Two quantities x and y are said to be in **direct proportion** if they increase (decrease) together in such a manner that the ratio of their corresponding values remains constant. That is if xy=k [k is a positive number, then x and y are said to vary directly. In such a case if y_1, y_2 are the values of y corresponding to the values x_1, x of x respectively then x1y1 = x2y2.
- If the number of articles purchased increases, the total cost also increases.
- More than money deposited in a bank, more is the interest earned.
- Quantities increasing or decreasing together need not always be in direct proportion, same in the case of inverse proportion.
- When two quantities x and y are in direct proportion (or vary directly), they are written as $x \propto y$. Symbol $\propto$stands for 'is proportion to'.
- **Inverse Proportion:** Two quantities x and y are said to be in **inverse proportion** if an increase in x causes a proportional decrease in y **(and vice-versa**) in such a manner that the product of their corresponding values remains constant. That is, if x y = k, then x and y are said to vary inversely. In this case if y_1, y_2 are the values of y corresponding to the values x_1, x_2 of x respectively then x_1, $Y_1 = x_2$, y_2 or x1y1 = x2y2
- When two quantities x and y are in inverse proportion (or vary inversely), they are written as $x \propto 1/y$. Example: If the number of workers increases, time taken to finish the job decreases. Or If the speed will increase the time required to cover a given distance will decrease.

CHAPTER - 13

Direct and Inverse Proportions

(Ex. 13.1)

1. Following are the car parking charges near a railway station up to

4 hours	Rs. 60
8 hours	Rs. 100
12 hours	Rs. 140
24 hours	Rs. 180

Check if the parking charges are in direct proportion to the parking time.

Sol. 460=115
8100=225
12140=335
24180=215
Hence, the parking charges are not in direct proportional to the parking time.

2. A mixture of paint is prepared by mixing 1 part of red pigments with 8 parts of base. In the following table, find the parts of base that need to be added.

Parts of red pigment	1	4	7	12	20
Parts of base	8	...	...	...	...

Sol. Let the ratio of parts of red pigment and parts of base be ab.
Here $a_1 = 1$, $b_1 = 8$
⇒a1b1=18 = k (say)
When $a_2 = 4$, $b_2 = ?$
k=a2b2 ⇒ b2=a2k=418=4×8 = 32
When $a_3 = 7_3$, b = ?
k=a3b3⇒ b3=a3k=718=7×8 = 56
When $a_4 = 12$, $b_4 = ?$
k=a4b4 ⇒ b4=a4k=1218=12×8 = 96
When $a_5 = 20$, $b_5 = ?$
k=a5b5 ⇒ b5=a5k=2018=20×8 = 160
Hence the table becomes,

Parts of red pigment	1	4	7	12	20
Parts of base	8	32	56	96	160

3. If 1 part of a red pigment requires 75 mL of base, how much red pigment should we mix with 1800 mL of base ?

Sol. Let the number of parts of red pigment be x and the amount of base be y mL.
As the number of parts of red pigment increases, amount of base also increases in the same ratio. It is a case of direct proportion. We make use of the relation of the type.

x1y1=x2y2
Here,
$x_1 = 1$
$y_1 = 75$ and $y_2 = 1800$
Therefore, x1y1=x2y2 gives
175=x21800
∴ $75x_2 = 1800$
∴ x2=180075
∴ $x_2 = 24$
Hence, 24 parts of the red pigment should be mixed.

4. A machine in a soft drink factory fills 840 bottles in six hours. How many bottles will it fill in five hours ?

Sol. Let it fill x bottles in five hours. We put the given information in the form of a table as shown below :

Number of bottles filled	840	x
Number of hours	6	5

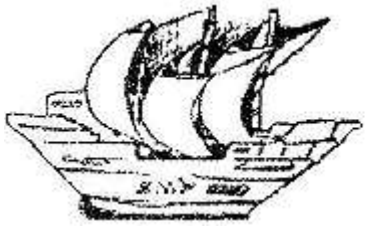
More the numbers of hours, more the number of bottles filled would be. So, the number of bottles filled and the number of hours are directly proportional to each other.
So, x1x2=y1y2
∴ 840x2=65
∴ $6x_2 = 840 \times 5$
∴ $x_2 = 840{\times}56$
∴ $x_2 = 700$
Hence, 700 bottles will be filled.

5. A photograph of a bacteria enlarged 50000 times attains a length of 5 cm as shown in the diagram. What is the actual length of the bacteria? If the photograph is enlarged 20000 times only, what be its enlarged length?

Sol. Actual length of the bacteria
550000cm
=110000cm.
$= 10^{-4}$ cm
More the number of times a photograph of a bacteria is enlarged, more the length attained. So, the number of times a photograph of a bacteria is enlarged and the length attained are directly proportional to each other.
So, x1x2=x2y2
∴ 500005=20000y2
∴ $50000\ y_2 = 5 \times 20000$
∴ y2=5×2000050000

∴ $y_2 = 2$
Hence, its enlarged length would be 2 cm.

6. In a model of a ship, the mast is 9 m high, while the mast of the actual ship is 12 m high. If the length of the ship is 28 m, how long is the model ship?



Sol. Let the length of the model ship be x m and the height of the mast be y cm.
We form a table as shown below:

Length of the ship (in metres)	28	x
Height of the mast (in metres)	12	9

More the length of the ship, more would be the length of its mast. Hence, this is a case of direct proportion. That is,
x1y1=x2y2
∴ 2812=x9
∴ $12x = 28 \times 9$
x=12×912
∴ $x = 21$
Hence, the length of the model ship is 21 m.

7(1). Suppose 2 kg of sugar contains 9×10^6 crystals. How many sugar crystals are there in 5 kg of sugar?

Sol. Let the number of sugar crystals in 5 kg of sugar be x.
The given information in the form of a table is as follows.

Amount of sugar (in kg)	2	5
Number of crystals	9×10^6	x

The amount of sugar and the number of crystals it contains are directly proportional to each other. Therefore, we obtain
29×106=5x
x=5×9×1062=2.25×107
Hence, the number of sugar crystals is 2.25×10^7.

7(2). Suppose 2 kg of sugar contains 9×10^6 crystals. How many sugar crystals are there in 1.2 kg of sugar?

Sol. Let x be the sugar crystals in 1.2 kg of sugar
The information given in the question can be formulated in a table as follows:

Sugar Weight	2 kg	1.2 kg
No. of crystals	9×10^6	x

The above information is in direct proportion:
Therefore,
$2(9\times106)=1.2x$
$x = 5.4 \times 10^6$ crystals

8. Rashmi has a road map with a scale of 1 cm representing 18 km. She drives on a road for 72 km. What would be her distance covered in the map ?

Sol. Let the distance covered in the map be x cm. Then,
1 : 18 = x : 72
∴ 118=x72
∴ x=7218
∴ x = 4
Hence, the distance covered in the map would be 4 cm.

9(1). A 5m 60cm high vertical pole casts a shadow 3m 20cm long. Find at the same time the length of the shadow cast by another pole 10m 50cm high.

Sol. Let the height of the vertical pole be xm and the length of the shadow by ym.
As the height of the vertical pole increases, the length of the shadow also increases in the same ratio, It is a case of direct proportion.
We make use of the relation of the type x1y1=x2y2.
Here,
x_1 = 5 m 60 cm = 5.60m
y_1 = 3 m 20 cm = 3.20m
x_2 = 10 m 50 cm = 10.50m
Therefore, x1y1=x2y2 gives
5.63.2=10.5y2
∴ $5.6y_2 = 3.2 \times 10.5$
∴ y2=3.2×10.55.6
∴ $y_2 = 6$
Hence, the length of the shadow is 6m.

9(2). A 5 m 60 cm high vertical pole casts a shadow 3 m 20 cm long. Find at the same time the height of a pole which casts a shadow 5m long.

Sol. Let the height of the vertical pole be x m and the length of the shadow by y m.
As the height of the vertical pole increases, the length of the shadow also increases in the same ratio, so it is a case of direct proportion.
We make use of the relation of the type x1y1=x2y2
Here,
x_1 = 5m 60cm = 560 cm
y_1 = 3m 20cm = 320 cm
x_2 = 5m 00cm = 500 cm

Therefore, x1y1=x2y2 gives
560320=x2500
∴ $320x_2 = 560 \times 500$
∴ x2=560×500320
∴ $x_2 = 875$ cm = 8m 75cm

10. A loaded truck travels 14 km in 25 minutes. If the speed remains the same, how for can it travels in 5 hours ?

Sol. Two quantities x and y which vary in direct proportion have the relation
x = k y or x y =k
Here, k= number of km it can travel time in hours
=14(2560)=14×6025
=1685
Now, x is the distance travelled in 5 hours
Using the relation x = ky, we obtain
x=1685×5
x = 168
Hence, it can tavel 168 km.

EX : 13.2

1(1). Is the number of workers on a job and the time to complete the job in inverse proportion ?

Sol. The number of workers on a jobs and the time to complete the job are in inverse proportion.

1(2). Is the time taken for a journey and the distance travelled in a uniform speed in inverse proportion ?

Sol. The time taken for a journey and the distance travelled in a uniform speed are not in inverse proportion.

1(3). Is the area of cultivated land and the crop harvested in inverse proportion ?

Sol. Area of cultivated land and the crop harvested are not in inverse proportion.

1(4). Is the time taken for a fixed journey and the speed of the vehicle in inverse proportion ?

Sol. The time taken for a fixed journey and the speed of the vechile are in inverse proportion.

1(5). Is the population of a country and the area of land per person are in inverse proportion ?

Sol. The population of a country and the area of land per person are in inverse proportion.

2. In a Television game show, the prize money of ₹1,00,000 is to be divided equally amongst the winners. Complete the table and find whether the prize money given to an individual winner is directly or inversely proportional to the number of winners:

No. of winners	1	2	4	5	8	10	20
Prize for each winner (in ₹)	1,00,000	50,000	...	...	...	...	...

Sol. Here the number of winners and prize money are in inverse proportion because when winners are increasing, prize money is decreasing.
When the number of winners are 4, each winner will get = 1000004 = Rs. 25,000
When the number of winners are 5, each winner will get = 1000005 = Rs. 20,000
When the number of winners are 8, each winner will get = 1000008 = Rs. 12,500
When the number of winners are 10, each winner will get = 10000010 = Rs. 10,000
When the number of winners are 20, each winner will get = 10000020 = Rs. 5,000

3(1). Rehman is making a wheel using spokes. He wants to fix equal spokes in such a way that the angles between any pair of consecutive spokes are equal. Help him by completing the following table.

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Numbers of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90°	60°	–	–	–

Are the number of spokes and the angles formed between the pairs of consecutive spokes in inverse proportion?

Sol. According to given information, the table can be completed as follows:

Numbers of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90°	60°	45°	36°	30°

Yes! The number of spokes and the angles formed between the pairs of consecutive spokes are in inverse proportion [$\because 4 \times 90° = 6 \times 60° = 8 \times 45° = 10 \times 36° = 12 \times 30°$]

3(2). Rehman is making a wheel using spokes. He wants to fix equal spokes in such a way that the angles between any pair of consecutive spokes are equal. Help him by completing the following table.

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Numbers of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90°	60°	–	–	–

Calculate the angle between a pair of consecutive spokes on a wheel with 15 spokes?

Sol. As per the information given:

Numbers of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90°	60°	45°	36°	30°

Let the angle between a pair of consecutive spokes on a wheel with 15 spokes be x°. Lesser the number of spokes, more will be the angle between a pair of consecutive spokes.
So, this is a case of inverse proportion.
Hence, $4\times90=15\times x$ [$\because x1y1=x2y2$]
$\therefore x=4\times9015$
$\therefore$ x = 24
Hence, the angle between a pair of consecutive spokes on a wheel with 15 spokes is 24°.

3(3). Rehman is making a wheel using spokes. He wants to fix equal spokes in such a way that the angles between any pair of consecutive spokes are equal. Help him by completing the following table.


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Numbers of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90°	60°	–	–	–

How many spokes would be needed, if the angle between a pair of consecutive spokes is 40°?

Sol. As per the information given, the table can be completed as follows:

Numbers of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90°	60°	45°	36°	30°

Let x spokes be needed
Lesser the number of spokes, more will be the angle between a pair of consecutive spokes.
So, this is a case of inverse proportion.
Hence, $4\times900=x\times400$ [$\because x_1y_1 = x_2y_2$]
$\therefore x=4\times90o40o$
$\therefore$ x = 9
Hence, 9 spokes would be needed for an angle of 40°

4. If a box of sweets is divided among 24 children, they will get 5 sweets each. How many would each get, if the number of the children is reduced by 4?

Sol. Suppose that each would get x sweets.
Thus, we have the following table.

Number of children	24	24 – 4 = 20
Number of sweets	5	x

Lesser the number of children, more will be the number of sweets each would get. So, this is a case of inverse proportion.
Hence, $24\times5=20\times x$
$\therefore x=24\times520$
$\therefore$ x = 6
Hence, each would get 6 sweets.

5. A farmer has enough food to feed 20 animals in his cattle for 6 days. How long would the food last if there were 10 more animals in his cattle ?

Sol. Suppose that the food would last for x days. We have the following table.

Number of animals	20	20 + 10 = 30
Number of days	6	x

We note that more the number of animals, lesser will be the number of days for which the food will last. Therefore, this is a case of inverse proportion.

So, $20\times6=30\times x$

$\therefore$ $x=20\times630$

$\therefore$ $x = 4$

Hence, the food would last for 4 days.

6. A contractor estimates that 3 persons could rewire Jasminder's house in 4 days. If, he uses 4 persons instead of three, how long should they take to complete the job ?

Sol. Suppose that they take x days to complete the job. We have the following table.

Number of persons	3	4
Number of days	4	x

More the number of persons, lesser will be the number of days required to complete the job. So, this is a case of inverse proportion.

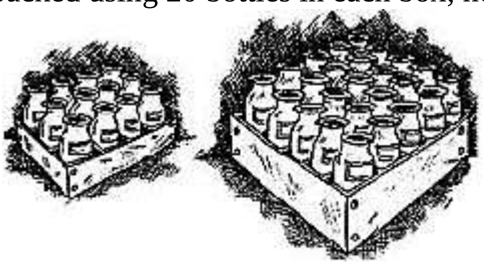
Hence, $3\times4=4\times x$

$\therefore$ $x=3\times44$

$\therefore$ $x = 3$

Hence, they would take 3 days to complete the job.

7. A batch of bottles were packed in 25 boxes with 12 bottles in each box. If the same batch is packed using 20 bottles in each box, how many boxes would be filled?



Sol. Suppose that x boxes would be filled. We have the following table.

Number of bottles	12	20
Number of boxes	25	x

Lesser the number of bottles more will be number of boxes required to be filled. So, this is a case of inverse proportion.

Hence, $12 \times 25 = 20 \times x$

$\therefore$ $x=12\times2520$

∴ x = 15

Hence, 15 boxes would be filled.

8. A factory requires 42 machines to produce a given number of articles in 63 days. How many machines would required to produce the same number of articles in 54 days ?

Sol. Suppose that x machines would be required. We have the following table.

Number of machines	42	x
Number of days	63	54

Lesser the number of machines, more will be the number of days to produce the same number of articles.

So, this is a case of inverse proportion.

Hence, 42 × 63 = x × 54

∴ x=42×6354

∴ x = 49

Hence, 49 machines would be required.

9. A car takes 2 hours to reach a destination by travelling at the speed of 60 km/h. How long will it take when the car travels at the speed of 80 km/h?

Sol. Let it take x hours. We have the following table.

Speed (in km/h)	60	80
Number of hours	2	x

Lesser the speed, more the number of hours to reach the destination.

Hence, 60 × 2 = 80 × x

∴ x=60×280

∴ x=32

∴ x=112

Thus, 112 hours would be taken.

10(1). Two persons could fit new windows in a house in 3 days. One of the persons fell ill before the work started. How long would the job take now?

Sol. Let the job would take x day.

We have the following table

Number of persons	2	2 – 1 = 1
Number of days	3	x

Clearly, less the number of persons, for more days they would do the job. So, the number of persons and number of days vary in inverse proportion

So, 2×3=1×x

∴ x = 6

Thus, the job would now take 6 days.

10(2). Two persons could fit new windows in a house in 3 days. How many persons would be needed to fit the windows in one day?

Sol. Let x persons be needed

We have the following table.

Number of days	3	1
Number of persons	2	x

Clearly, more the number of persons, faster would they do the job. So, the number of persons and number of days vary in inverse proportion.

So, $3\times2=1\times x$

$\therefore x = 6$

Thus, 6 persons would be needed.

11. A school has 8 periods a day each of 45 minutes duration. How long would each period be, if the school has 9 periods a day, assuming the number of school hours to be the same ?

Sol. Let each period be x minutes long.

We have the following table

Number of periods	8	9
Length of each period (in minutes)	45	x

We note that more the number of periods, lesser would be the length of each period. Therefore, this is a case of inverse proportion.

So, $=8\times45=9\times x$

$\therefore x=8\times459$

$\therefore x = 40$

Hence, each period would be 40 minutes long.